Effect of Transition Waveguides on Dielectric Waveguide Directional Couplers

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Abstract —**The coupling of energy between two curved dielectric waveguides is investigated by the staircase approximation method, which combines the building block approach of multimode network theory with a rigorous mode-matching procedure. Particular attention is directed toward two major effects of transition waveguides on the performance of directional couplers composed of dielectric waveguides: one is the change in the coupling length, the other is the radiation loss. The coupling problem is analyzed in terms of the scattering of an incident guided mode by the coupler structure as a whole. Numerical results are given to illustrate the coupling characteristics of various structures and to establish useful guidelines for the design of directional couplers.**

I. Introduction

THE PROBLEM of energy coupling between two parallel uniform dielectric waveguides with transition regions has been investigated in the literature. Because of the open nature of dielectric waveguides, a directional coupler requires that the input/output waveguides be kept at a distance to be isolated from each other, while a region of relatively strong coupling is provided with two waveguides sufficiently close to each other. Thus, it is necessary to introduce transition waveguides to connect the input/output waveguides with the coupling region [1]. The main purpose of the transition region is to provide a smooth flow of energy from the input/output waveguide region into the coupling region, or vice versa. Intuitively, it is expected that the transition regions should be as gradual as possible, so that the scattering losses can be reduced. On the other hand, the length of the transition regions may be limited in an integrated circuit environment, and it is necessary to determine the effect of the transition on the general scattering characteristics of the system. Also, the transition regions may provide additional coupling and it is important to assess their effect in order to establish an effective coupling length of the directional coupler as a whole.

Because of the mathematical difficulties involved in an analysis of a curved waveguide, particularly of the open type, the effect of the transition waveguides has been considered under restrictive conditions. In the past, it has been commonly assumed that the transition length is sufficiently large, so that the radiation and scattering effect caused by the curved structures can be neglected; certain analyses did not even consider the end effect or the reflection and directivity caused by coupling in the curved sections.

We present an analysis of the effect of the transition waveguides from the viewpoint of guided mode scattering by a nonuniform coupler structure. Such a method has been shown to be particularly powerful for determining the effect of the transition profile. In this analysis, two approximations are introduced. One is the staircase approximation of the continuous transition profile; this is a discretization in geometry. Evidently, in the limit of vanishing step size the piecewise-constant profile will approach the continuous one. Another approximation is an enclosure of the whole structure inside an oversize parallel-plate waveguide, so that the modal spectrum is discretized to facilitate the analysis. With such approximations, the mathematical analysis and the physical interpretation of wave phenomena associated with nonuniform dielectric waveguides can be kept simple and clear. It should be stressed that although the discretization of modes does introduce some degree of approximation, as far as the surface waves are concerned, the presence of an oversize parallel-plate waveguide does not change the physics of the problem.

With the piecewise-constant profile the structure can be viewed as consisting of uniform waveguides and junctions. The fields can be represented by the complete set of waveguide modes for each uniform region and are then required to satisfy the boundary conditions at each junction. The method of mode matching is then employed to solve the scattering problem including not only the fundamental mode of interest but also the effect of all the higher order modes.

Since the variation of the scattered power of the fundamental mode, including the reflected and transmitted power, means the conversion of power into the higher order modes, this is interpreted as radiation into the continuous spectrum in the case of an open structure. Therefore, in the present approach the radiated power is given

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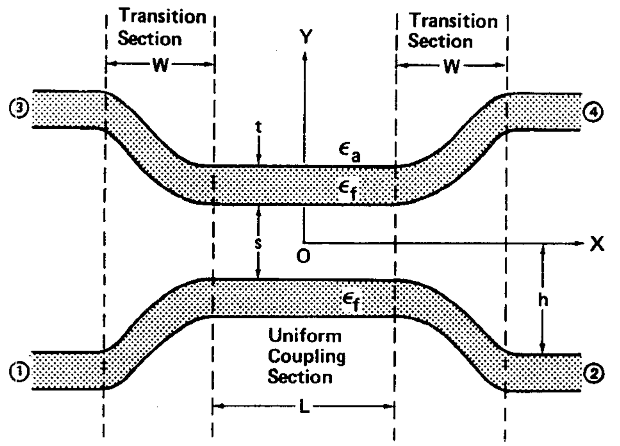

Fig. 1. Structure configuration of parallel dielectric waveguide directional coupler.

by the total power of the higher order modes of the partially filled parallel-plate waveguide. Because the structure under consideration is lossless, the total power must be conserved, and such a conservation condition is used as a gauge of our numerical accuracy.

Finally, numerical results are presented to quantify the two major effects of the transition waveguides: the radiation loss due to the curvature of the waveguides and the change in the coupling length of the structure as a whole. On the basis of these numerical results, useful guidelines are developed for the design of directional couplers composed of dielectric waveguide.

II. Method of Analysis

Fig. 1 shows the geometrical configuration of the dielectric waveguide directional coupler under investigation. It consists of one uniform coupling region which is connected to uniform input/output waveguides by curved transition sections. In the present study, we assume that the center line of the transition profile is characterized by a hyperbolic tangent function, as was previously done [3]–[5]. The structure is symmetric with respect to both the x and the y axis. In this paper, only this special geometry is considered for simplicity; other geometrics can be handled in similar fashion, although the computations will be more complicated. Utilizing the symmetry property, we may bisect the structure in both x and y directions and consider only a quarter of the structure with appropriate boundary conditions, as shown in Fig. 2. The stepped structure can be viewed as consisting of a series of uniform, partially filled parallel-plate waveguides connected by junctions. A basic equivalent network has been developed for a junction between two uniform dielectric waveguides [3]. The basic equivalent network for all the junctions can be put in cascade through transmission line sections to form an overall network for the entire stepped dielectric structure. The scattering of the guided mode by the staircase structure can be analyzed in terms of the reflection characteristics by each basic unit.

Fig. 3 depicts the ith basic unit that consists of the ith step discontinuity between two uniform waveguides at the point $x = x_i$ and the uniform waveguide of length l_i between the points x_i and x_{i-1}. The analysis procedure of the reflection characteristic for the basic unit is outlined below.

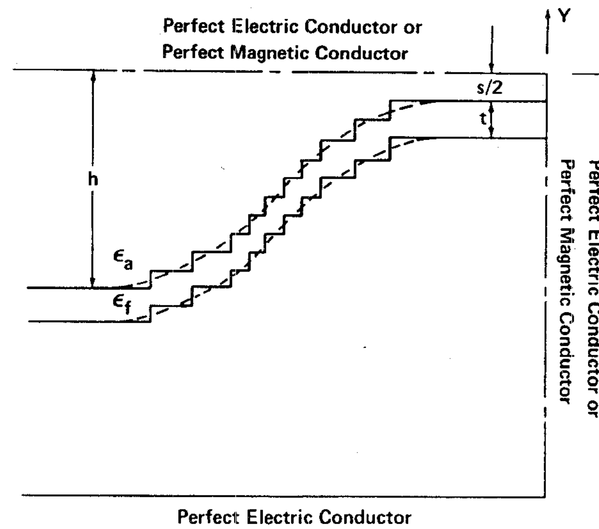

Fig. 2. Bisected structure with boundary conditions.

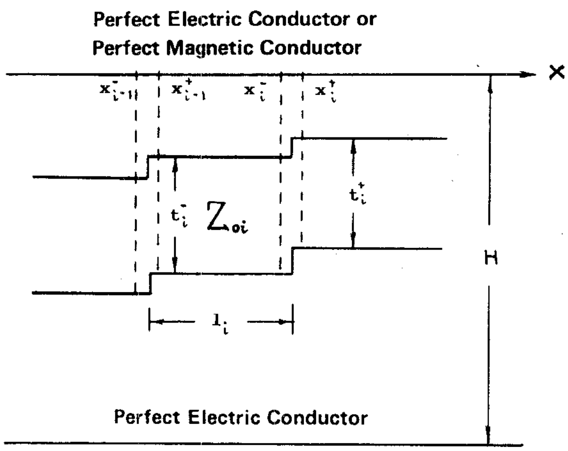

Fig. 3. The ith basic unit of the stepped structure.

Since the terminal plane of the stepped structure at $x = 0$ is known to be either a perfect electric wall or a perfect magnetic wall, the input impedance matrix $Z(x_i^+)$ at the $x = x_i^+$ plane looking to the right can be determined by the impedance transform technique. Therefore, $Z(x_i^+)$ is considered as a known matrix for the ith basic unit. It is well known [6] that the electromagnetic fields in each uniform waveguide section can be represented by a complete set of mode functions; for a TE polarized mode in the region with the dielectric layer of thickness t_i^+, we have the tangential field components represented by

$$E_z^{(i)}(x, y) = \sum_n V_n(t_i^+, x)\phi_n(t_i^+, y) \tag{1}$$

$$H_y^{(i)}(x, y) = \sum_n I_n(t_i^+, x)\phi_n(t_i^+, y) \tag{2}$$

where $\phi_n(t_i^+, y)$ is the nth mode function and $V_n(t_i^+, x)$ and $I_n(t_i^+, x)$ are the voltage and the current of the nth mode. It is noted that throughout this paper, the summation over n extends from 1 to ∞. By matching the tangential field components at the ith discontinuity, the waveguide problem is reduced to the analysis of an equivalent network for the modal voltages and currents which are on the two sides of the ith step and are related by

$$V_m(t_i^-, x_i^-) = \sum_n (Q_i)_{mn} V_n(t_i^+, x_i^+) \quad (3)$$

$$I_m(t_i^-, x_i^-) = \sum_n (Q_i)_{mn} I_n(t_i^+, x_i^+) \quad (4)$$

where $(Q_i)_{mn}$ is the coefficient of coupling between the mth mode on the left and the nth mode on the right of the ith step discontinuity and is defined by the scalar product

$$(Q_i)_{mn} = \langle \phi_m(t_i^-, y) | \phi_n(t_i^+, y) \rangle = \int_0^H \phi_m(t_i^-, y) \phi_n(t_i^+, y)\, dy \quad (5)$$

where ϕ_m and ϕ_n are, respectively, the mode functions in the uniform dielectric waveguides. They are the solutions of the Sturm–Liouville eigenvalue problem [6].

Equations (3) and (4) may be written in matrix form as

$$V(x_i^-) = Q_i V(x_i^+) \quad (6)$$

$$I(x_i^-) = Q_i I(x_i^+) \quad (7)$$

where $V(x_i^-)$ and $I(x_i^-)$ are column vectors with the transmission line voltage and current of the TE mode on the left of the ith step, and $V(x_i^+)$ and $I(x_i^+)$ are column voltage and current vectors of the TE mode on the right of the ith step. Since the voltage and current are related as

$$V(x_i^-) = Z(x_i^-) I(x_i^-) \quad (8)$$

$$V(x_i^+) = Z(x_i^+) I(x_i^+) \quad (9)$$

(6) and (7) may be expressed by

$$Z(x_i^-) = Q_i Z(x_i^+) Q_i^{-1}. \quad (10)$$

We assume that the dielectric materials forming the waveguide in Fig. 1 are lossless. The Sturm–Liouville eigenvalue problem of the multilayer planar dielectric structure is Hermitian, because of the perfectly electric or magnetic conducting bounding plates at $y = 0$ and $-H$. Therefore, all eigenvalues are real and all eigenmode functions can be chosen to be real, and we have $Q_i^{-1} = Q_{it}$. Then (10) can be written as

$$Z(x_i^-) = Q_i Z(x_i^+) Q_{it} \quad (11)$$

where t stands for the transpose, and Q_i is the coupling coefficient matrix of the ith step whose elements are determined by (5).

From (11), the reflection coefficient matrix $\Gamma(x_i^-)$, at the $x = x_i^-$ plane looking to the right, can easily be obtained as

$$\Gamma(x_i^-) = [Z(x_i^-) + Z_{0i}]^{-1}[Z(x_i^-) - Z_{0i}]. \quad (12)$$

Then, the input impedance matrix at the $x = x_{i-1}^-$ plane looking to the right is determined by the impedance transform technique as

$$Z(x_{i-1}^+) = Z_{0i}[I + H_i\Gamma(x_i^-)H_i][I - H_i\Gamma(x_i^-)H_i]^{-1} \quad (13)$$

where Z_{0i} and H_i are, respectively, the characteristic impedance and the phase matrices of the ith step discontinuity. They are all diagonal matrices, and their elements are

$$(Z_{0i})_{mn} = \delta_{mn} K_{xin} / \omega\epsilon_0\epsilon_{ein} \quad (14)$$

$$(H_i)_{mn} = \delta_{mn} \exp(-jK_{xin} l_i) \quad (15)$$

where δ_{mn} stands for the Kronecker delta, K_{xin} is the propagation wavenumber in the x direction for the nth mode in the dielectric waveguide of the ith section, and ϵ_{ein} is the effective dielectric constant of the nth mode. Thus, the reflection coefficient matrix of any step junction can be determined by using (5) and (11)–(15).

With the bisections in both the x and the y direction, there are four different combinations of boundary conditions. For an incident guided mode from waveguide 1, we analyze the four separate substructures, as depicted by Fig. 2, with appropriate boundary conditions. In each case, the energy is totally reflected, and the reflection coefficient matrices, accounting for the coupling to higher order modes, are denoted by R_{ss}, R_{sa}, R_{as}, and R_{aa} for the four cases. The subscripts s and a stand for the symmetric and antisymmetric bisections, respectively; the first subscript represents the symmetry property with respect to the x axis and the second with respect to the y axis. The scattering coefficient matrices of the overall structure are found to be given by

$$G_1 = (R_{ss} + R_{sa} + R_{as} + R_{aa})/4.0 \quad (16)$$

$$T_2 = (R_{ss} - R_{sa} + R_{as} - R_{aa})/4.0 \quad (17)$$

$$T_3 = (R_{ss} + R_{sa} - R_{as} - R_{aa})/4.0 \quad (18)$$

$$T_4 = (R_{ss} - R_{sa} - R_{as} + R_{aa})/4.0. \quad (19)$$

Here G_1 is the reflection coefficient matrix of waveguide 1, and T_2, T_3, and T_4 are the transmission coefficient matrices from waveguide 1 to waveguides 2, 3, and 4, respectively. The scattering of an incident guided mode is then simply determined for the coupling structure by (16)–(19).

III. Numerical Results

Referring to Fig. 1, we assume that each individual waveguide supports only the fundamental guided mode. A fundamental guided mode of unit power is incident from the left into waveguide 1. We are interested in the effect of the transition sections on the reflection of the fundamental mode in waveguide 1 and the transmission of the same mode into all other waveguides. While the individual waveguides support only a single mode, the coupled waveguide system is designed to support two surface wave modes, one symmetric and the other antisymmetric. The two modes propagate with different phase velocities; consequently, they interfere with each other constructively or destructively. This results in the transfer of energy back and forth between the two coupled waveguides.

Fig. 4 shows the effect of the transition length on the scattered powers of the fundamental mode in each waveguide and the total radiated power from the entire nonuniform dielectric structure, which represents the coupling to

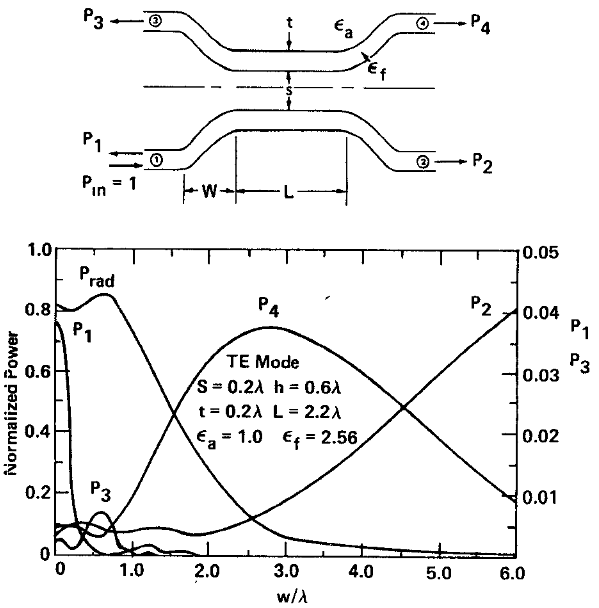

Fig. 4. Dependence of scattered powers on transition length.

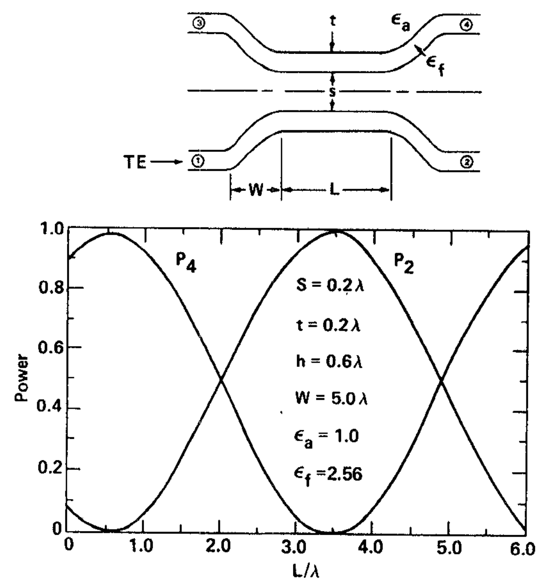

Fig. 5. Dependence of transmitted power on the length on the uniform coupling region.

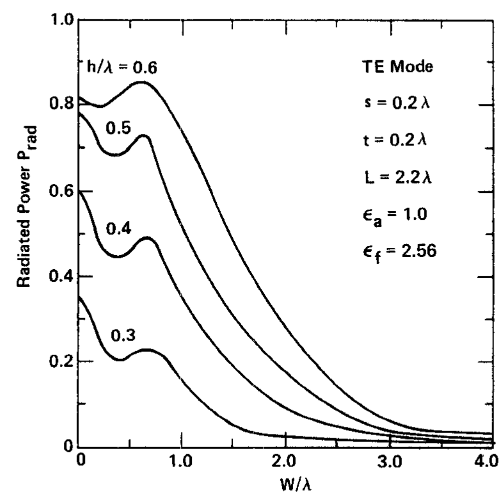

Fig. 6. Effect of the transition length on the radiated power for the parallel dielectric waveguide coupler.

the higher order modes. The length of the uniform coupling region is fixed at $L = 2.2\lambda$, where λ is the free-space wavelength. It is noted that the scale for the reflected power P_1, and reverse-coupled power P_3, is exaggerated, as shown on the right-hand side of the figure. They are quite small and can be neglected as long as the transitions are not too short. For a fixed length of the coupling region L, the transition regions have two major effects: one is the radiation loss and the other is the change in the effective coupling length. For the case analyzed, the radiation loss exceeds 80 percent of the incident power for a transition length of less than one wavelength. When the transition length is increased, the radiation loss decreases rapidly and the division of the transmitted power between waveguides 2 and 4 varies continually. For the length of the uniform coupling region, $L = 2.2\lambda$, it is expected that a very long transition length will be required to realize a coupling close to 100 percent to either waveguide 2 or 4. For a reasonable transition length, it is expected that we may adjust the length of the uniform coupling region to achieve any desired ratio of the powers between waveguides 2 and 4, as discussed next.

Fig. 5 shows the transmission characteristics of the coupler, with the structure parameters depicted in the inset. The coupling length over which a complete power transfer takes place is determined by

$$L_{\text{eff}} = \lambda / 4.0 \, |n_e - n_o| \tag{20}$$

where n_e and n_o are effective indices of the first even and the first odd mode of the uniform dielectric waveguide, respectively. For a 3 dB coupling, the coupling length is one half of the value of L_{eff} given above. For the case shown, the coupling length is observed to be about 2.9λ, and the 3 dB coupling length is about 1.45λ. Evidently, 100 percent coupling is possible and any ratio of powers between waveguides 2 and 4 can be achieved.

From Figs. 4 and 6, we observe that the radiation loss is the main problem for the dielectric waveguide directional coupler with curved transition sections if the transition length is small. It is expected intuitively that when the transition length or the radius of curvature of the bend is sufficiently large, the radiation loss will become small. The question is how large a transition length is needed to make the radiation loss negligible or tolerable. This is what we intend to illustrate. Fig. 6 shows the effect of the transition length on the radiated power, P_{rad}, for four values of the separation distance h. It is found that as long as the transition length $w > 5\lambda$, the radiated power can be kept below 2 percent of the incident power, and it becomes insensitive to the separation distance h and the transition length w. Fig. 6 also shows that, for small w, the larger the separation distance h, the stronger the radiation. This is because of the stronger fields near the discontinuities for larger h. These facts establish a quantitative basis for the

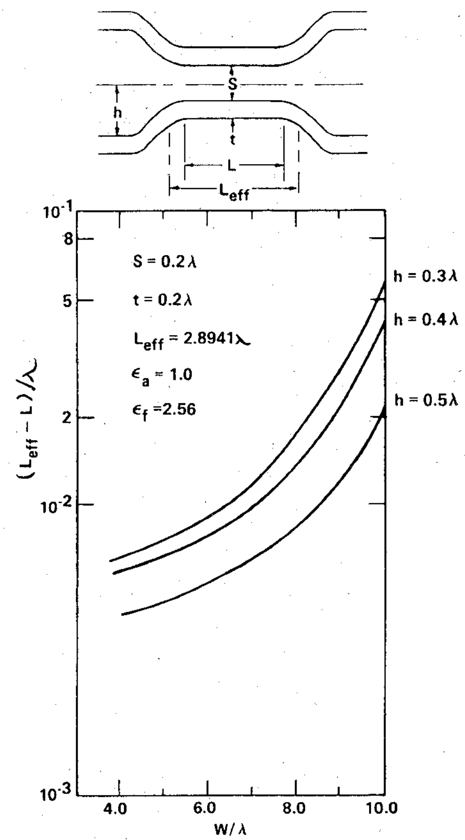

Fig. 7. Effect of the transition length on the coupling length for the parallel dielectric waveguide coupler.

design of dielectric waveguide directional couplers, as far as the undesirable radiation is concerned.

The curves in Fig. 7 show the effect of the transition length on the effective coupling length of a 3 dB coupler for several values of the separation distance h. As depicted in the inset, the effective coupling length includes the effect of the curved section, in addition to the uniform coupling region. In other words, we may define a correction length as the difference between the effective coupling length and the actual length of the uniform coupling region. Here, we observe that the longer the transition length w, the larger the correction length $(L_{\mathrm{eff}} - L)$. Furthermore, for the same transition length, the correction length becomes smaller when the separation of input/output waveguides increases. The reason for this is that the larger the separation, the steeper the curved sections and the weaker the coupling in the transition region. From the curves in Fig. 7, we found that the correction to the coupling length becomes quite significant when the transition length is large. Therefore, while increasing the transition length to reduce the scattering losses, it is important to keep in mind that the coupling length has to be adjusted accordingly.

IV. Conclusion

We have investigated the coupling of the guided modes on curved dielectric waveguide directional couplers by the staircase approximation method, which combines the building block approach of multimode network theory with a rigorous mode-matching procedure. Extensive numerical data have been obtained to quantify the propagation and scattering characteristics of the coupling structure. The power of the fundamental mode in each input/output waveguide is carefully examined for the parallel dielectric waveguide coupler. It is found that the reflection, reverse coupling, and radiation decrease rapidly as the transition length is increased; they are practically negligible for a transition length greater than five wavelengths. For a long transition region, it is shown that the coupling length has to be adjusted to account for the additional coupling of the transition regions. Based on these results, some useful guidelines for the design of the coupling structures are thereby suggested.

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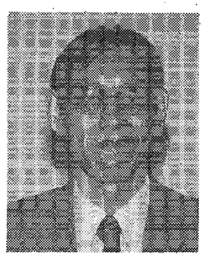

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